

1 Determinante

1. Izračunati vrijednost determinati

$$\begin{aligned}
 & \text{(a)} \begin{vmatrix} 2 & 4 \\ -3 & 9 \end{vmatrix}; \text{ (b)} \begin{vmatrix} 2+2i & 3-i \\ -3+4i & -1-i \end{vmatrix}; \text{ (c)} \begin{vmatrix} \frac{x^2+1}{1-x^2} & \frac{2x}{1-x^2} \\ \frac{2x}{1-x^2} & \frac{x^2+1}{1-x^2} \end{vmatrix}; \\
 & \text{(d)} \begin{vmatrix} \sin x + \sin y & \cos x + \cos y \\ \cos y - \cos x & \sin x - \sin y \end{vmatrix}; \text{ (e)} \begin{vmatrix} 1 & \log_a b \\ \log_b a & 1 \end{vmatrix}; \\
 & \text{(f)} \begin{vmatrix} 5 & 6 & 3 \\ 0 & 1 & 2 \\ 7 & 4 & 5 \end{vmatrix}; \text{ (g)} \begin{vmatrix} 1 & 2 & 0 \\ -3 & 2 & 9 \\ -1 & 2 & 1 \end{vmatrix}; \text{ (h)} \begin{vmatrix} a & a & a \\ -a & a & x \\ -a & -a & x \end{vmatrix}; \text{ (i)} \begin{vmatrix} 1 & i & 1+i \\ -i & 1 & 0 \\ 1-i & 0 & 1 \end{vmatrix}; \\
 & \text{(j)} \begin{vmatrix} 3 & 1 & 2 & 3 \\ 4 & -1 & 2 & 4 \\ 1 & -1 & 1 & 1 \\ 4 & -1 & 1 & 5 \end{vmatrix}; \text{ (k)} \begin{vmatrix} 1 & 1 & 3 & 4 \\ 2 & 0 & 0 & 8 \\ 3 & 0 & 0 & 2 \\ 4 & 4 & 7 & 5 \end{vmatrix}; \text{ (l)} \begin{vmatrix} 3 & 1 & 2 & 3 \\ 4 & -1 & 2 & 4 \\ 1 & -1 & 1 & 1 \\ 4 & -1 & 1 & 5 \end{vmatrix}; \\
 & \text{(m)} \begin{vmatrix} 7 & 2 & 1 & 3 & 4 \\ 1 & 0 & 2 & 0 & 3 \\ 3 & 0 & 4 & 0 & 7 \\ 6 & 3 & 2 & 4 & 5 \\ 5 & 1 & 2 & 2 & 3 \end{vmatrix}; \text{ (n)} \begin{vmatrix} x & 0 & -1 & 1 & 0 \\ 1 & x & -1 & 1 & 0 \\ 1 & 0 & x-1 & 0 & 1 \\ 0 & 1 & -1 & x & 1 \\ 0 & 1 & -1 & 0 & x \end{vmatrix}.
 \end{aligned}$$

2. Pokazati da je

$$\begin{aligned}
 & \text{(a)} \begin{vmatrix} b^2+c^2 & ab & ca \\ ab & c^2+a^2 & bc \\ ca & bc & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2; \text{ (b)} \begin{vmatrix} 1 & a & a^2+a^3 \\ 1 & a^2 & a^3+a \\ 1 & a^3 & a+a^2 \end{vmatrix} = 0; \\
 & \text{(c)} \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & b \end{vmatrix} = (1-a)(1-b); \text{ (d)} \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = (b-c)(b-a)(a-c); \\
 & \text{(e)} \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (b-a)(c-a)(c-b); \text{ (f)} \begin{vmatrix} ax & a^2+x^2 & 1 \\ ay & a^2+y^2 & 1 \\ az & a^2+z^2 & 1 \end{vmatrix} = a(x-y)(x-z)(z-y); \\
 & \text{(g)} \begin{vmatrix} 1 & bc & b+c \\ 1 & ac & a+c \\ 1 & ab & a+b \end{vmatrix} = (a-b)(b-c)(c-a); \text{ (h)} \begin{vmatrix} a-b & 2a & 2a \\ 2b & b-a & 2b \\ a-b & 2a & a-b \end{vmatrix} = (a+b)^3; \\
 & \text{(i)} \begin{vmatrix} a+b & -a & -b \\ -b & b+c & -c \\ -a & -c & c+a \end{vmatrix} = 0; \text{ (j)} \begin{vmatrix} -x & y & z & 1 \\ x & -y & z & 1 \\ x & y & -z & 1 \\ x & y & z & -1 \end{vmatrix} = -8xyz; \\
 & \text{(k)} \begin{vmatrix} a & b & c & d \\ a & -b & -c & -d \\ a & b & -c & -d \\ a & b & c & -d \end{vmatrix} = -8abcd; \text{ (l)} \begin{vmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{vmatrix} = -a(a-b)(c-b)(d-c).
 \end{aligned}$$

3. Izračunati

$$\begin{aligned}
 & \text{(a)} \begin{vmatrix} a^{-4} & a^{-3} & a^{-2} \\ a^{-1} & 1 & a \\ a^2 & a^3 & a^4 \end{vmatrix} (=0); \text{ (b)} \begin{vmatrix} a & b & a & 1 \\ b & a & b & 1 \\ a & -a & b & 1 \\ b & -b & a & 1 \end{vmatrix} (=2(a+b)(b-a)^2); \\
 & \text{(c)} \begin{vmatrix} 1 & 1 & 1 \\ 1 & z & z^2 \\ 1 & z^2 & z \end{vmatrix}, \text{ ako je } z = \cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}; \text{ (d)} \begin{vmatrix} 1 & 1 & z \\ 1 & 1 & z^2 \\ z^2 & z & 1 \end{vmatrix}, \text{ ako je } z = \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3};
 \end{aligned}$$

$$(e) \begin{vmatrix} \sin 2x & -\cos 2x & 1 \\ \sin x & -\cos x & \cos x \\ \cos x & \sin x & \sin x \end{vmatrix}; (= 0) \quad (f) \begin{vmatrix} 1 + \cos x & 1 + \sin x & 1 \\ 1 - \sin x & 1 + \cos x & 1 \\ 1 & 1 & 1 \end{vmatrix} (= 1).$$

4. Riješiti jednačine

$$(a) \begin{vmatrix} x & 2 & 3 \\ 3 & 1 & 2 \\ 1 & 3 & 4 \end{vmatrix} = 0; \quad (b) \begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix} = 0; \quad (c) \begin{vmatrix} \log_c x & \log_c x - n \\ \log_c x - m & \log_c x \end{vmatrix} = 0, \quad (0 < c \neq 1);$$

$$(d) \begin{vmatrix} 1 & 1 & 2 & 3 \\ 1 & 2 - x^2 & 2 & 3 \\ 2 & 3 & 1 & 5 \\ 2 & 3 & 1 & 9 - x^2 \end{vmatrix} = 0; \quad (e) \begin{vmatrix} x - 3 & x + 2 & x - 1 \\ x + 2 & x - 4 & x \\ x - 1 & x + 4 & x - 5 \end{vmatrix} = 0;$$

$$(f) \begin{vmatrix} \sin\left(x + \frac{\pi}{4}\right) & \sin x & \cos x \\ \sin\left(x + \frac{\pi}{4}\right) & \cos x & \sin x \\ 1 & a & 1 - a \end{vmatrix} = \frac{\sqrt{2} - 2}{4}; \quad (g) \begin{vmatrix} 1 & 1 & 2 & 3 \\ 1 & 2 - x^2 & 2 & 3 \\ 2 & 3 & 1 & 5 \\ 2 & 3 & 1 & 1 - x^2 \end{vmatrix} = 0.$$

5. Riješiti nejednačine

$$(a) \begin{vmatrix} x & -1 & 0 \\ 5 & -1 & -6 \\ -1 & 0 & x \end{vmatrix} \geq 0; \quad (b) \begin{vmatrix} 1 & 0 & x \\ 0 & 1 & 1 \\ 1 & x & 0 \end{vmatrix} \leq \begin{vmatrix} x & 3 & x \\ 2 & 1 & 3 \\ 1 & x & 1 \end{vmatrix}.$$