

1 Sistemi linearnih jednačina

1. Ispitati saglasnost sistema, i u slučaju saglasnosti riješiti sisteme

$$\begin{array}{lll}
 \begin{array}{l} x + 2y + 4z = 7 \\ -3x + 2y + 4z = 3 \\ 2x + y + z = 4 \\ 3x + 3y - 3z = 3; \end{array} & \begin{array}{l} x + 2y + 3z = 6 \\ 2x - y - z = 1 \\ x + y + 4z = 6 \\ 3x + y + z = 5; \end{array} & \begin{array}{l} x - y + 3z = 3 \\ 2x - 2y + 6z = 6 \\ 3x + 4y + 5z = 12 \\ -x - 2y + z = -2; \end{array} \\
 \text{(a)} & \text{(b)} & \text{(c)}
 \end{array}$$

$$\begin{array}{lll}
 \begin{array}{l} x + y + z = 0 \\ 2x - y - z = 0 \\ 4x + y + z = 0; \end{array} & \begin{array}{l} x + 2y = -3 \\ -x + 3z = 9 \\ x + y + 2z = 1; \end{array} & \begin{array}{l} x - y + z = 2 \\ x + 2y + 4z = 5 \\ 3x + 2y + z = 4 \\ -2x - 2y + z = -1; \end{array} \\
 \text{(d)} & \text{(e)} & \text{(f)}
 \end{array}$$

$$\begin{array}{lll}
 \begin{array}{l} 2x + y + 3z = 1 \\ x - 5y + z = -10 \\ 4x + y + z = 5 \\ x + y - z = 4; \end{array} & \begin{array}{l} x - y + z = 0 \\ x + 2y + 4z = 6 \\ 3x + 2y + z = 3 \\ -2x - 2y + z = -1; \end{array} & \begin{array}{l} x + 3y + z = 0 \\ 3x + y - 2z = -2 \\ x + y + z = 2 \\ y + 5z = 9; \end{array} \\
 \text{(g)} & \text{(h)} & \text{(i)}
 \end{array}$$

$$\begin{array}{l} 2x + y + 3z = 1 \\ x - 5y + z = -10 \\ 4x + y + z = 5 \\ x + y - z = 4; \end{array}$$

(j)

$$\begin{array}{lll}
 \begin{array}{l} x + 2y + 3z - 2t = 4 \\ 2x + y - z + 5t = 7 \\ x - y - z + 6t = 5 \\ 3x + 3y + 2z + 3t = 11; \end{array} & \begin{array}{l} x + 3y - z + t = 4 \\ 2x - y + 2z + 3t = 6 \\ 4x + y + 5z - 2t = 8 \\ 3x + 2y + z + 4t = 10; \end{array} & \begin{array}{l} 2x - y + 4z - t = 4 \\ x + 3y - 2z = 2 \\ 3x + y + z - 2t = 3 \\ 3x + 2y + 2z - t = 6; \end{array} \\
 \text{(k)} & \text{(l)} & \text{(m)}
 \end{array}$$

$$\begin{array}{lll}
 \begin{array}{l} 2x - y + 4z - t = 4 \\ -x + 7y - 6z + t = 1 \\ 3x + z - 5t = -1 \\ 5x - y + 5z - 6t = 3; \end{array} & \begin{array}{l} 3x - 2y + z - 2t = 0 \\ -2x + 4y + 3z + 2t = 7 \\ -x + 3z - t = 1 \\ 2x - 2y + 4z - 3t = 1; \end{array} & \begin{array}{l} 2x - 3y + z - t = -1 \\ -2x + z + 3t = 2 \\ -x - y - 3z + 5t = 0 \\ 6x - 2y - z + t = 4; \end{array} \\
 \text{(n)} & \text{(o)} & \text{(p)}
 \end{array}$$

$$\begin{array}{lll}
 \begin{array}{l} 2x + 7y + 3z + t = 6 \\ 3x + 5y + 2z = 4 \\ 9x + 4y + z + 7t = 2; \end{array} & \begin{array}{l} x + 2y - 3z + t = 1 \\ x + y + z + t = 2 \\ -x + 3y + z - t = 0; \end{array} & \begin{array}{l} x + y + z + 2t = 5 \\ 2x + y - z + t = 1 \\ 4x + 3y + z + 3t = 11; \end{array} \\
 \text{(q)} & \text{(r)} & \text{(s)}
 \end{array}$$

$$\begin{array}{lll}
 \begin{array}{l} 2x - 3y + 4z - t = 6 \\ -x + 2z + 3t = 1 \\ -9x + 2y + z - 2t = -8; \end{array} & \begin{array}{l} 2x + 7y + 3z + t = 6 \\ 3x + 5y + 2z = 4 \\ 9x + 4y + z + 7t = 2. \end{array} & \begin{array}{l} 3x - 2y + 6z - 5t = 2 \\ x + 3y - 4z + t = 1 \\ 2x - 5y + 10z - 6t = 1; \end{array} \\
 \text{(t)} & \text{(u)} & \text{(v)}
 \end{array}$$

2. Izvršiti diskusiju rješenja datih sistema u zavisnosti od vrijednosti parametara koji se javljaju u njima

$$\begin{array}{llll}
 \begin{array}{l} ax + y - z = 1 \\ x + ay - z = 1; \\ x - y - az = 1 \end{array} & \begin{array}{l} ax + y + z = 1 \\ x + ay + z = a; \\ x + y + az = a^2 \end{array} & \begin{array}{l} x + y + z = a \\ x + (a + 1)y + z = 2a; \\ x + y + (a + 1)z = 0 \end{array} & \begin{array}{l} x + y + z = 6 \\ ax + 4y + z = 5 \\ 6x + (a + 2)y + 2z = 13. \end{array} \\
 \text{(a)} & \text{(b)} & \text{(c)} & \text{(d)}
 \end{array}$$

3. Ispitati da li homogeni sistemi imaju netrivialna rješenja i u slučaju da imaju izračunati ih

$$\begin{array}{llllll}
 \begin{array}{l} x - y + z = 0 \\ x + 3y + z = 0; \\ 3x + y - z = 0 \end{array} & \begin{array}{l} x - y - z = 0 \\ x + 3y + z = 0; \\ 3x + y - z = 0 \end{array} & \begin{array}{l} 2x - 3y + z = 0 \\ 3x - y + 2z = 0; \\ 3x + y - 2z = 0 \end{array} & \begin{array}{l} x - y + z = 0 \\ 2x - 2y + 2z = 0; \\ 3x - 3y + 3z = 0 \end{array} & \begin{array}{l} x + y + z = 0 \\ 3x - y + 2z = 0; \\ x - 3y = 0. \end{array} \\
 \text{(a)} & \text{(b)} & \text{(c)} & \text{(d)} & \text{(e)}
 \end{array}$$

4. Odrediti vrijednost parametra koji se pojavljuje u sistemima, tako da sistem ima i netrivialna rješenja, a zatim riješiti sistem

$$\begin{array}{lll}
 \begin{array}{l} x - y + az = 0 \\ x + 3y + z = 0; \\ 3x + y - z = 0 \end{array} & \begin{array}{l} ax + 3y + 2z = 0 \\ 2x + y + 3z = 0; \\ -3x + 4y + z = 0 \end{array} & \begin{array}{l} 3x - y + z = 0 \\ ax - y + 2z = 0 \\ x + ay + (a + 1)z = 0. \end{array} \\
 \text{(a)} & \text{(b)} & \text{(c)}
 \end{array}$$